Content Area: Mathematics
Grade Level Expectations: Fourth Grade
Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates: (Click on a Prepared Graduate Competency to View Articulated Expectations)

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities.

Concepts and skills students master:
1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms.

Evidence Outcomes

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<tr>
<th>Students Can:</th>
<th>21st Century Skill and Readiness Competencies</th>
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<tr>
<td>a. Generalize place value understanding for multi-digit whole numbers (CCSS: 4.NBT)</td>
<td>Inquiry Questions:</td>
</tr>
<tr>
<td>i. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. (CCSS: 4.NBT.1)</td>
<td>1. Why isn’t there a “oneths” place in decimal fractions?</td>
</tr>
<tr>
<td>ii. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. (CCSS: 4.NBT.2)</td>
<td>2. How can a number with greater decimal digits be less than one with fewer decimal digits?</td>
</tr>
<tr>
<td>iii. Compare two multi-digit numbers based on meanings of the digits in each place, using &gt;, =, and &lt; symbols to record the results of comparisons. (CCSS: 4.NBT.2)</td>
<td>3. Is there a decimal closest to one? Why?</td>
</tr>
<tr>
<td>iv. Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.3)</td>
<td>Relevance &amp; Application:</td>
</tr>
<tr>
<td>b. Use decimal notation to express fractions, and compare decimal fractions (CCSS: 4.NF)</td>
<td>1. Decimal place value is the basis of the monetary system and provides information about how much items cost, how much change should be returned, or the amount of savings that has accumulated.</td>
</tr>
<tr>
<td>i. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and</td>
<td>2. Knowledge and use of place value for large numbers provides context for population, distance between cities or landmarks, and attendance at events.</td>
</tr>
</tbody>
</table>

Nature Of:
1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.

2. Mathematicians reason abstractly and quantitatively. (MP)

3. Mathematicians look for and make use of structure. (MP)

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1. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. (CCSS: 4.NF.6)

2. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. (CCSS: 4.NF.6)

3. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. (CCSS: 4.NF.7)
**Prepared Graduates:** *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

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**Concepts and skills students master:**
2. Different models and representations can be used to compare fractional parts

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<td><strong>Students Can:</strong></td>
<td><strong>Inquiry Questions:</strong></td>
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a. Use ideas of fraction equivalence and ordering to: (CCSS: 4.NF)
   i. Explain equivalence of fractions using drawings and models.⁴
   ii. Use the principle of fraction equivalence to recognize and generate equivalent fractions. (CCSS: 4.NF.1)
   iii. Compare two fractions with different numerators and different denominators,⁵ and justify the conclusions.⁶ (CCSS: 4.NF.2)

b. Build fractions from unit fractions by applying understandings of operations on whole numbers. (CCSS: 4.NF)
   i. Apply previous understandings of addition and subtraction to add and subtract fractions.⁷
      1. Compose and decompose fractions as sums and differences of fractions with the same denominator in more than one way and justify with visual models.
      2. Add and subtract mixed numbers with like denominators.⁸ (CCSS: 4.NF.3c)
      3. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.⁹ (CCSS: 4.NF.3d)
   ii. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (CCSS: 4.NF.4)
      1. Express a fraction a/b as a multiple of 1/b.¹⁰ (CCSS: 4.NF.4a)
      2. Use a visual fraction model to express a/b as a

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**Inquiry Questions:**
1. How can different fractions represent the same quantity?
2. How are fractions used as models?
3. Why are fractions so useful?
4. What would the world be like without fractions?

**Relevance & Application:**
1. Fractions and decimals are used any time there is a need to apportion such as sharing food, cooking, making savings plans, creating art projects, timing in music, or portioning supplies.
2. Fractions are used to represent the chance that an event will occur such as randomly selecting a certain color of shirt or the probability of a certain player scoring a soccer goal.
3. Fractions are used to measure quantities between whole units such as number of meters between houses, the height of a student, or the diameter of the moon.

**Nature Of:**
1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.
2. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
3. Mathematicians model with mathematics. (MP)
multiple of $1/b$, and apply to multiplication of whole number by a fraction.\textsuperscript{11} (CCSS: 4.NF.4b)

3. Solve word problems involving multiplication of a fraction by a whole number.\textsuperscript{12} (CCSS: 4.NF.4c)

\textsuperscript{4} Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. (CCSS: 4.NF.1)

\textsuperscript{5} e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, (CCSS: 4.NF.2)

\textsuperscript{6} e.g., by using a visual fraction model. (CCSS: 4.NF.2)

\textsuperscript{7} Understand a fraction $a/b$ with $a > 1$ as a sum of fractions $1/b$. (CCSS: 4.NF.3)
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (CCSS: 4.NF.3a)
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$. (CCSS: 4.NF.3b)

\textsuperscript{8} e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (CCSS: 4.NF.3c)

\textsuperscript{9} e.g., by using visual fraction models and equations to represent the problem. (CCSS: 4.NF.3d)

\textsuperscript{10} For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$. (CCSS: 4.NF.4a)

\textsuperscript{11} For example, $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.) (CCSS: 4.NF.4b)

\textsuperscript{12} e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? (CCSS: 4.NF.4c)
Prepared Graduates: (Click on a Prepared Graduate Competency to View Articulated Expectations)

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Concepts and skills students master:
3. Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency

Evidence Outcomes | 21st Century Skill and Readiness Competencies
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**Students Can:**

a. Use place value understanding and properties of operations to perform multi-digit arithmetic. (CCSS: 4.NBT)
   i. Fluently add and subtract multi-digit whole numbers using standard algorithms. (CCSS: 4.NBT.4)
   ii. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. (CCSS: 4.NBT.5)
   iii. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS: 4.NBT.6)
   iv. Illustrate and explain multiplication and division calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.6)

b. Use the four operations with whole numbers to solve problems. (CCSS: 4.OA)
   i. Interpret a multiplication equation as a comparison.\(^\text{13}\) (CCSS: 4.OA.1)
   ii. Represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS: 4.OA.1)
   iii. Multiply or divide to solve word problems involving multiplicative comparison.\(^\text{14}\) (CCSS: 4.OA.2)
   iv. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders

**Inquiry Questions:**

1. Is it possible to make multiplication and division of large numbers easy?
2. What do remainders mean and how are they used?
3. When is the “correct” answer not the most useful answer?

**Relevance & Application:**

1. Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra.

**Nature Of:**

1. Mathematicians envision and test strategies for solving problems.
2. Mathematicians develop simple procedures to express complex mathematical concepts.
3. Mathematicians make sense of problems and persevere in solving them. (MP)
4. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
5. Mathematicians look for and express regularity in repeated reasoning. (MP)
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<td>v.</td>
<td>Represent multistep word problems with equations using a variable to represent the unknown quantity. (CCSS: 4.OA.3)</td>
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<tr>
<td>vi.</td>
<td>Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.3)</td>
</tr>
<tr>
<td>vii.</td>
<td>Using the four operations analyze the relationship between choice and opportunity cost (PFL).</td>
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</table>

3 e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. (CCSS: 4.OA.1)

4 e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (CCSS: 4.OA.2)
Content Area: Mathematics  
Grade Level Expectations: Fourth Grade  
Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates: (Click on a Prepared Graduate Competency to View Articulated Expectations)

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Concepts and skills students master:  
1. Number patterns and relationships can be represented by symbols

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| a. Generate and analyze patterns and identify apparent features of the pattern that were not explicit in the rule itself.  
  (CCSS: 4.OA.5)  
  i. Use number relationships to find the missing number in a sequence  
  ii. Use a symbol to represent and find an unknown quantity in a problem situation  
  iii. Complete input/output tables  
  iv. Find the unknown in simple equations  
| 1. What characteristics can be used to classify numbers into different groups?  
  2. How can we predict the next element in a pattern?  
  3. Why do we use symbols to represent missing numbers?  
  4. Why is finding an unknown quantity important? |
| b. Apply concepts of squares, primes, composites, factors, and multiples to solve problems  
  i. Find all factor pairs for a whole number in the range 1–100. (CCSS: 4.OA.4)  
  ii. Recognize that a whole number is a multiple of each of its factors. (CCSS: 4.OA.4)  
  iii. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. (CCSS: 4.OA.4)  
  iv. Determine whether a given whole number in the range 1–100 is prime or composite. (CCSS: 4.OA.4) | Relevance & Application:  
  1. Use of an input/output table helps to make predictions in everyday contexts such as the number of beads needed to make multiple bracelets or number of inches of expected growth.  
  2. Symbols help to represent situations from everyday life with simple equations such as finding how much additional money is needed to buy a skateboard, determining the number of players missing from a soccer team, or calculating the number of students absent from school.  
  3. Comprehension of the relationships between primes, composites, multiples, and factors develop number sense. The relationships are used to simplify computations with large numbers, algebraic expressions, and division problems, and to find common denominators. |

Nature Of:

1. Mathematics involves pattern seeking.
2. Mathematicians use patterns to simplify calculations.
3. Mathematicians model with mathematics. (MP)

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For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. (CCSS: 4.OA.5)
Content Area: Mathematics
Grade Level Expectations: Fourth Grade
Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Concepts and skills students master:
1. Visual displays are used to represent data

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<tr>
<td>a. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). (CCSS: 4.MD.4)</td>
<td>1. What can you learn by collecting data?</td>
</tr>
<tr>
<td>b. Solve problems involving addition and subtraction of fractions by using information presented in line plots.¹ (CCSS: 4.MD.4)</td>
<td>2. What can the shape of data in a display tell you?</td>
</tr>
</tbody>
</table>

Inquiry Questions:
1. What can you learn by collecting data?
2. What can the shape of data in a display tell you?

Relevance & Application:
1. The collection and analysis of data provides understanding of how things work. For example, measuring the weather every day for a year helps to better understand weather.

Nature Of:
1. Mathematics helps people use data to learn about the world.
2. Mathematicians model with mathematics. (MP)
3. Mathematicians use appropriate tools strategically. (MP)
4. Mathematicians attend to precision. (MP)

¹ For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. (CCSS: 4.MD.4)
Content Area: Mathematics  
Grade Level Expectations: Fourth Grade  
Standard: 4. Shape, Dimension, and Geometric Relationships

**Prepared Graduates:** *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

**Concepts and skills students master:**
1. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time

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<td><strong>Students Can:</strong></td>
<td><strong>Inquiry Questions:</strong></td>
</tr>
<tr>
<td>a. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (CCSS: 4.MD)</td>
<td>1. How do you decide when close is close enough?</td>
</tr>
<tr>
<td>i. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. (CCSS: 4.MD.1)</td>
<td>2. How can you describe the size of geometric figures?</td>
</tr>
<tr>
<td>ii. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.¹ (CCSS: 4.MD.1)</td>
<td><strong>Relevance &amp; Application:</strong></td>
</tr>
<tr>
<td>iii. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. (CCSS: 4.MD.2)</td>
<td>1. Accurate use of measurement tools allows people to create and design projects around the home or in the community such as flower beds for a garden, fencing for the yard, wallpaper for a room, or a frame for a picture.</td>
</tr>
<tr>
<td>iv. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS: 4.MD.2)</td>
<td><strong>Nature Of:</strong></td>
</tr>
<tr>
<td>v. Apply the area and perimeter formulas for rectangles</td>
<td>1. People use measurement systems to specify the attributes of objects with enough precision to allow collaboration in production and trade.</td>
</tr>
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</table>

¹. Inquiry Questions:
2. Relevance & Application:
3. Nature Of:
b. Use concepts of angle and measure angles. (CCSS: 4.MD)
   i. Describe angles as geometric shapes that are formed wherever two rays share a common endpoint, and explain concepts of angle measurement. (CCSS: 4.MD.5)
   ii. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (CCSS: 4.MD.6)
   iii. Demonstrate that angle measure is additive. (CCSS: 4.MD.7)
   iv. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems. (CCSS: 4.MD.7)

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1 For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ... (CCSS: 4.MD.1)

2 For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. (CCSS: 4.MD.3)

3 An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. (CCSS: 4.MD.5a)
   An angle that turns through $n$ one-degree angles is said to have an angle measure of $n$ degrees. (CCSS: 4.MD.5b)

4 When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. (CCSS: 4.MD.7)

5 e.g., by using an equation with a symbol for the unknown angle measure. (CCSS: 4.MD.7)
### Prepared Graduates: *(Click on a Prepared Graduate Competency to View Articulated Expectations)*

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

### Concepts and skills students master:

2. Geometric figures in the plane and in space are described and analyzed by their attributes

### Evidence Outcomes | 21st Century Skill and Readiness Competencies
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**Students Can:** | **Inquiry Questions:**

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<tr>
<th>Students Can:</th>
<th>1. How do geometric relationships help us solve problems?</th>
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<tbody>
<tr>
<td>a. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. (CCSS: 4.G.1)</td>
<td>2. Is a square still a square if it’s tilted on its side?</td>
</tr>
<tr>
<td>b. Identify points, line segments, angles, and perpendicular and parallel lines in two-dimensional figures. (CCSS: 4.G.1)</td>
<td>3. How are three-dimensional shapes different from two-dimensional shapes?</td>
</tr>
<tr>
<td>c. Classify and identify two-dimensional figures according to attributes of line relationships or angle size.⁶ (CCSS: 4.G.2)</td>
<td>4. What would life be like in a two-dimensional world?</td>
</tr>
<tr>
<td>d. Identify a line of symmetry for a two-dimensional figure.⁷ (CCSS: 4.G.3)</td>
<td>5. Why is it helpful to classify things like angles or shapes?</td>
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### Inquiry Questions:

1. How do geometric relationships help us solve problems?
2. Is a square still a square if it’s tilted on its side?
3. How are three-dimensional shapes different from two-dimensional shapes?
4. What would life be like in a two-dimensional world?
5. Why is it helpful to classify things like angles or shapes?

### Relevance & Application:

1. The understanding and use of spatial relationships helps to predict the result of motions such as how articles can be laid out in a newspaper, what a room will look like if the furniture is rearranged, or knowing whether a door can still be opened if a refrigerator is repositioned.
2. The application of spatial relationships of parallel and perpendicular lines aid in creation and building. For example, hanging a picture to be level, building windows that are square, or sewing a straight seam.

### Nature Of:

1. Geometry is a system that can be used to model the world around us or to model imaginary worlds.
2. Mathematicians look for and make use of structure. (MP)
3. Mathematicians look for and express regularity in repeated reasoning. (MP)

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⁶ Based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (CCSS: 4.G.2)

⁷ As a line across the figure such that the figure can be folded along the line into matching parts. (CCSS: 4.G.3)

Identify line-symmetric figures and draw lines of symmetry. (CCSS: 4.G.3)